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# On the rotational invariance and non-invariance of lepton angular distributions in Drell–Yan and quarkonium production

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## ABSTRACT

Several rotational invariant quantities for the lepton angular distributions in Drell–Yan and quarkonium production were derived several years ago, allowing the comparison between different experiments adopting different reference frames. Using an intuitive picture for describing the lepton angular distribution in these processes, we show how the rotational invariance of these quantities can be obtained. This approach can also be used to determine the rotational invariance or non-invariance of various quantities specifying the amount of violation for the Lam–Tung relation. While the violation of the Lam–Tung relation is often expressed by frame-dependent quantities, we note that alternative frame-independent quantities are preferred.

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The angular distributions of leptons produced in the Drell–Yan process [1] and the quarkonium production in hadron–hadron collisions [2,3] remain a subject of considerable interest. The polar and azimuthal angular distributions of leptons produced in unpolarized and polarized Drell–Yan process allow the extraction of various types of transverse-momentum dependent distributions [4,5]. First (leading order) results on the extraction of the Boer–Mulders functions [6,7] have been obtained from unpolarized Drell–Yan experiments using pion [8,9] or proton [10] beams, indicating that the quark transverse spin is correlated with the quark transverse momentum inside unpolarized protons. A more precise determination of the amount of quark polarization requires inclusion of higher order perturbative corrections because gluon radiation can also affect the lepton angular distributions [11–14]. Recent measurement of Drell–Yan angular distributions with a pion beam on a transversely polarized proton target provided the first information from Drell–Yan on the correlation between the quark transverse momentum and the spin direction of a transversely polarized proton [15]. For quarkonium production, the lepton angular distributions reveal sensitively the underlying partonic mechanisms,

as various subprocesses could lead to distinct polarizations for the quarkonium [3,16,17].

The lepton angular distributions in Drell–Yan and quarkonium production are generally measured in the rest frame of the dileptons. Many different choices of the reference frames exist in the literature, depending on how the axes of the coordinate system are chosen. While it is common to define the  $y$  axis to be along the direction normal to the reaction plane (which is the plane containing the beam axis and the dilepton's momentum vector) and the  $x$  and  $z$  axes lying on the reaction plane, the specific direction of the  $z$  axis is chosen differently for different reference frames. In particular, the Collins–Soper frame [18] has the  $z$  axis bisecting the beam and target momentum vectors, while the helicity frame aligns the  $z$  axis with the dilepton momentum vector in the center-of-mass frame. The Gottfried–Jackson frame [19] and the  $u$ -channel frame have the  $z$  axis parallel to the beam and target momentum direction, respectively. These various reference frames are related to each other by rotations along the  $y$  axis by certain angles [8,20].

A general expression for the lepton angular distribution in the Drell–Yan process or quarkonium production is given as

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \lambda_\phi \sin^2 \theta \cos 2\phi, \quad (1)$$

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where  $\theta$  and  $\phi$  refer to the polar and azimuthal angles of  $l^-$  ( $e^-$  or  $\mu^-$ ) in the rest frame of the dilepton. While the polar angle dependence is specified by the parameter  $\lambda_\theta$ , the azimuthal dependencies of the lepton angular distributions are described by the parameters  $\lambda_{\theta\phi}$  and  $\lambda_\phi$ . Note that these parameters are related to the parameters  $\lambda, \mu, \nu$  in Ref. [21] as  $\lambda_\theta = \lambda$ ,  $\lambda_{\theta\phi} = \mu$  and  $\lambda_\phi = \nu/2$ . The values of  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$  depend on the choice of the coordinate system. While the Collins–Soper frame is chosen by many experiments for the data analysis, other reference frames are also utilized by some experiments. Going from one frame to another acts as a nonlinear transformation on these three parameters [22], making it hard to connect the results in different frames.

The frame-dependence of the angular distribution parameters could potentially lead to confusion when comparing results of lepton angular distributions or quarkonium polarizations measured in different experiments [20,23]. In order to mitigate the confusion caused by the frame dependence of the parameters  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$ , Faccioli et al. [24–26] pointed out that various quantities can be formed from  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$  with the property that they are invariant under the transformations among different reference frames. The comparison between measurements obtained with different reference frames could be performed, if such rotation invariant quantities are used rather than the individual  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$  parameters. Examples of such rotational invariant quantities include [25,26]

$$\mathcal{F} = \frac{1 + \lambda_\theta + 2\lambda_\phi}{3 + \lambda_\theta}, \quad (2)$$

and

$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}. \quad (3)$$

The reason for considering these particular combinations is not just the rotational invariance, but also that they are measures for the deviation of the Lam–Tung relation [27],  $1 - \lambda_\theta = 4\lambda_\phi$ , that is satisfied in the Drell–Yan process at order  $\alpha_s$  in case of collinear parton distributions. Its violation results from the acoplanarity of the partonic subprocess, as discussed in detail in Refs. [12,14]. This acoplanarity can arise from intrinsic transverse momentum of quarks inside the proton, but also from perturbative gluon radiation beyond order  $\alpha_s$ . They lead to a deviation of  $\mathcal{F}$  from  $\frac{1}{2}$  and of  $\tilde{\lambda}$  from 1. In contrast, the deviation of  $1 - \lambda_\theta - 4\lambda_\phi$  from zero often considered in experimental and theoretical studies [7–11] is *not* a rotationally invariant quantity, pointed out first in Ref. [26], and hence a potential source of confusion when comparing its values obtained in different frames.

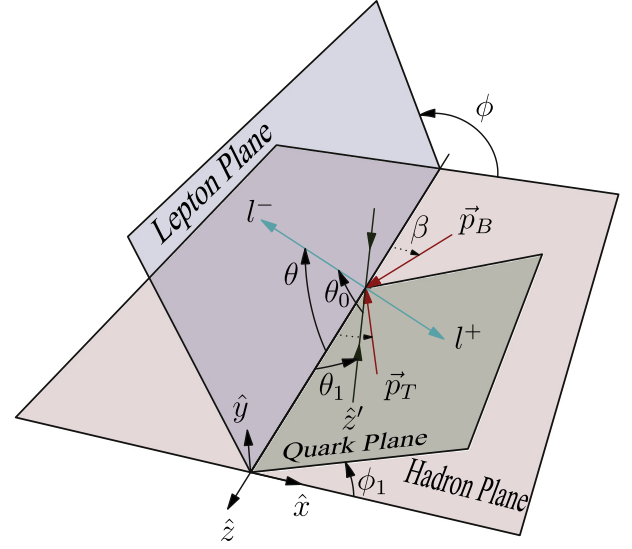
Another rotation-invariant quantity invoking all three parameters is [3,28]

$$\tilde{\lambda}' = \frac{(\lambda_\theta - \lambda_\phi)^2 + 4\lambda_{\theta\phi}^2}{(3 + \lambda_\theta)^2}. \quad (4)$$

Although not immediately obvious from their definition in terms of  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$ , the above three quantities,  $\mathcal{F}$ ,  $\tilde{\lambda}$ ,  $\tilde{\lambda}'$ , are invariant only under rotations around the  $y$  axis, which includes the transformations connecting the various reference frames in the literature. On the other hand, the quantity  $\mathcal{G}$  is invariant under the rotation along the  $x$  axis [26],

$$\mathcal{G} = \frac{1 + \lambda_\theta - 2\lambda_\phi}{3 + \lambda_\theta}. \quad (5)$$

Finally,  $\lambda_\theta$  is invariant under the rotation along the  $z$ -axis [26].



**Fig. 1.** Definition of the Collins–Soper frame and various angles and planes in the rest frame of  $\gamma^*/Z$  or a vector quarkonium. The hadron plane is formed by  $\vec{p}_B$  and  $\vec{p}_T$ , the momentum vectors of the two interacting hadrons. The  $\hat{x}$  and  $\hat{z}$  axes of the Collins–Soper frame both lie in the hadron plane with  $\hat{z}$  axis bisecting the  $\vec{p}_B$  and  $-\vec{p}_T$  vectors. The quark ( $q$ ) and antiquark ( $\bar{q}$ ) annihilate collinearly with equal momenta to form  $\gamma^*/Z$  or a vector quarkonium, while the quark momentum vector  $\hat{z}'$  and the  $\hat{z}$  axis form the quark plane. The polar and azimuthal angles of  $\hat{z}'$  in the Collins–Soper frame are  $\theta_1$  and  $\phi_1$ . The  $l^-$  and  $l^+$  are emitted back-to-back with  $\theta$  and  $\phi$  as the polar and azimuthal angles for  $l^-$ .

The rotational invariance of  $\mathcal{F}$ ,  $\tilde{\lambda}$ ,  $\tilde{\lambda}'$  and  $\mathcal{G}$  was obtained in Refs. [25,26,28,29] from the consideration of the covariance properties of angular momentum eigenstates of a vector meson. In a recent study [12,14], it was shown that some salient features of the parameters  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$  in the Drell–Yan process and  $Z$ -boson production can be well described by an intuitive approach. In particular, the pronounced transverse-momentum dependence of  $\lambda_\theta$  and  $\lambda_\phi$  for  $Z$ -boson production and the clear violation of the Lam–Tung relation at the LHC [30,31] can be well described by this approach. In this paper, we show how the rotational invariance properties of  $\mathcal{F}$ ,  $\tilde{\lambda}$ ,  $\tilde{\lambda}'$  and  $\mathcal{G}$  can be deduced using the approach of Refs. [12,14]. It is also clear from the analysis below that the rotational invariance or non-invariance of various quantities characterizing the violation of the Lam–Tung relation can be obtained.

In the dilepton rest frame, we first define three different planes, namely, the hadron plane, the quark plane, and the lepton plane, shown in Fig. 1. For dileptons with non-zero transverse momentum,  $q_T$ , the momenta of the two interacting hadrons,  $\vec{p}_B$  and  $\vec{p}_T$ , are not collinear in the rest frame of  $\gamma^*/Z$ , and they form the “hadron plane” shown in Fig. 1. Fig. 1 also shows the “lepton plane”, formed by the momentum vector of the  $l^-$  and the  $\hat{z}$  axis. In the rest frame of the dilepton, the  $l^-$  and  $l^+$  are emitted back-to-back with equal momenta.

In the dilepton rest frame, a pair of collinear  $q$  and  $\bar{q}$  with equal momenta annihilate into a  $\gamma^*/Z$  or a vector quarkonium, as illustrated in Fig. 1. We define the momentum unit vector of  $q$  as  $\hat{z}'$ , and the “quark plane” is formed by the  $\hat{z}'$  and  $\hat{z}$  axes. The polar and azimuthal angles of the  $\hat{z}'$  axis in the Collins–Soper frame are denoted as  $\theta_1$  and  $\phi_1$ . For the coplanar case,  $\phi_1 = 0$  and the hadron plane coincides with the quark plane. When  $\phi_1 \neq 0$ ,  $\phi_1$  signifies the acoplanarity angle. With respect to the  $q - \bar{q}$  axis, called the natural axis [32], the  $l^-$  has an azimuthally symmetric angular distribution, namely,

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \lambda_0 \cos^2 \theta_0, \quad (6)$$

where  $\theta_0$  is the polar angle between the  $l^-$  momentum vector and the  $\hat{z}'$  axis (see Fig. 1), and  $a$  is the forward-backward asymmetry originating from the parity-violating coupling, which is important only when the dilepton mass is close to the  $Z$  boson mass. The parameter  $\lambda_0$  depends on the reaction mechanism. For Drell-Yan process in which a virtual photon decays into a lepton pair, we have  $\lambda_0 = 1$ . This is a consequence of helicity conservation leading to a transversely polarized virtual photon with respect to the natural axis. For quarkonium production, the value of  $\lambda_0$  depends on the specific mechanism. We note that  $\lambda_0 = 0$  for unpolarized quarkonium production, while  $\lambda_0 = -1$  for production of longitudinally polarized quarkonium.

The angles  $\theta$  and  $\phi$  are experimental observables, and it is necessary to express  $\theta_0$  in terms of  $\theta$  and  $\phi$ . This can be accomplished using the following trigonometric relation:

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1). \quad (7)$$

Substituting Eq. (7) into Eq. (6), we obtain

$$\begin{aligned} \frac{d\sigma}{d\Omega} \propto & (1 + \frac{1}{2}\lambda_0 \sin^2 \theta_1) + (\lambda_0 - \frac{3}{2}\lambda_0 \sin^2 \theta_1) \cos^2 \theta \\ & + (\frac{1}{2}\lambda_0 \sin 2\theta_1 \cos \phi_1) \sin 2\theta \cos \phi \\ & + (\frac{1}{2}\lambda_0 \sin^2 \theta_1 \cos 2\phi_1) \sin^2 \theta \cos 2\phi \\ & + (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\ & + (\frac{1}{2}\lambda_0 \sin^2 \theta_1 \sin 2\phi_1) \sin^2 \theta \sin 2\phi \\ & + (\frac{1}{2}\lambda_0 \sin 2\theta_1 \sin \phi_1) \sin 2\theta \sin \phi \\ & + (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi. \end{aligned} \quad (8)$$

A comparison between Eq. (1) and Eq. (8) shows that  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$ , and  $\lambda_\phi$  can be expressed as a function of  $\lambda_0$ ,  $\theta_1$  and  $\phi_1$  (cf. with [32] for zero acoplanarity angle  $\phi_1 = 0$ ):

$$\begin{aligned} \lambda_\theta &= \frac{2\lambda_0 - 3\lambda_0 \sin^2 \theta_1}{2 + \lambda_0 \sin^2 \theta_1} \\ \lambda_{\theta\phi} &= \frac{\lambda_0 \sin 2\theta_1 \cos \phi_1}{2 + \lambda_0 \sin^2 \theta_1} \\ \lambda_\phi &= \frac{\lambda_0 \sin^2 \theta_1 \cos 2\phi_1}{2 + \lambda_0 \sin^2 \theta_1}. \end{aligned} \quad (9)$$

The terms proportional to  $\sin 2\phi$  and  $\sin \phi$  do not appear due to Lorentz invariance, provided there are no vectors (like transverse polarization) normal to the hadron plane. Such terms in Eq. (8) integrate to zero due to the acoplanarity angle average. Unless one considers polarized leptons, parity or time-reversal violation, Eq. (8) reduces to Eq. (1).

First, we consider the quantity  $\mathcal{F}$  in Eq. (2). From Eq. (9), we obtain

$$\mathcal{F} = \frac{1 + \lambda_0 - 2\lambda_0 \sin^2 \theta_1 \sin^2 \phi_1}{3 + \lambda_0} = \frac{1 + \lambda_0 - 2\lambda_0 y_1^2}{3 + \lambda_0}, \quad (10)$$

where  $y_1 = \sin \theta_1 \sin \phi_1$  is the component of the unit vector  $\hat{z}'$  along the  $y$ -axis in the dilepton rest frame. The invariance of  $\mathcal{F}$  with respect to a rotation along the  $y$  axis is clearly shown in Eq. (10), since  $\lambda_0$  and  $y_1$  are both invariant under such a rotation. It is interesting to note that for the Drell-Yan process, where  $\lambda_0 = 1$ ,  $\mathcal{F}$  becomes  $(1 - y_1^2)/2$ . As pointed out in Refs. [12,14],  $y_1$ , or the non-coplanarity angle  $\phi_1$  between the hadron and the quark planes in Fig. 1, is in general not equal to zero. For the special case of  $\phi_1 = 0$  (or  $y_1 = 0$ ),  $\mathcal{F} = 1/2$  and  $\mathcal{F}$  is invariant under

any arbitrary rotation in the dilepton's rest frame. As discussed in Refs. [12,14], the Lam-Tung relation in the Drell-Yan process is satisfied when the angle  $\phi_1$  vanishes. This is verified from Eq. (9), when the values of  $\lambda_0$  and  $\phi_1$  are set at 1 and 0, respectively.

We next consider the quantity  $\tilde{\lambda}$ . Using Eq. (9), Eq. (3) becomes

$$\tilde{\lambda} = \frac{\lambda_0 + 3\lambda_0 \sin^2 \theta_1 \sin^2 \phi_1}{1 + \lambda_0 \sin^2 \theta_1 \sin^2 \phi_1} = \frac{\lambda_0 + 3\lambda_0 y_1^2}{1 + \lambda_0 y_1^2}. \quad (11)$$

Again,  $\tilde{\lambda}$  must be invariant under a rotation along the  $y$  axis, since  $\lambda_0$  and  $y_1$  are both invariant under such rotation. In the special case of coplanarity between the hadron plane and the quark plane, we have  $y_1 = 0$ , and Eq. (11) becomes  $\tilde{\lambda} = \lambda_0$ . In that case,  $\tilde{\lambda}$  is invariant under rotation along any axis. However,  $\tilde{\lambda}$  is in general not the same as  $\lambda_0$ , and  $\tilde{\lambda}$  is in general not invariant under an arbitrary rotation.

We turn our attention next to the quantity  $\tilde{\lambda}'$  in Eq. (4). All three parameters,  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$ , and  $\lambda_\phi$  are involved in  $\tilde{\lambda}'$ . Using Eq. (9), we obtain

$$\tilde{\lambda}' = \frac{\lambda_0^2 (z_1^2 + x_1^2)^2}{(3 + \lambda_0)^2} = \frac{\lambda_0^2 (1 - y_1^2)^2}{(3 + \lambda_0)^2}, \quad (12)$$

where  $z_1$  is the component of the unit vector  $\hat{z}'$  along the  $z$  axis and the identity  $x_1^2 + y_1^2 + z_1^2 = 1$  is used. Thus,  $\tilde{\lambda}'$  is invariant under a rotation along the  $y$  axis. For the coplanar case,  $y_1 = 0$  and  $\tilde{\lambda}'$  is invariant under rotation along any axis.

In an analogous fashion, one can show the invariance of  $\mathcal{G}$  and  $\lambda_\theta$  under the rotation along the  $x$  and  $z$  axis, respectively. Using Eq. (9), Eq. (5) becomes

$$\mathcal{G} = \frac{1 + \lambda_0 - 2\lambda_0 \sin^2 \theta_1 \cos^2 \phi_1}{3 + \lambda_0} = \frac{1 + \lambda_0 - 2\lambda_0 x_1^2}{3 + \lambda_0}, \quad (13)$$

where  $x_1 = \sin \theta_1 \cos \phi_1$  is the component of the unit vector  $\hat{z}'$  along the  $x$  axis in the dilepton rest frame. Similarly, from Eq. (9), the parameter  $\lambda_\theta$  can be written as

$$\lambda_\theta = \frac{-\lambda_0 + 3\lambda_0 \cos^2 \theta_1}{2 + \lambda_0 - \lambda_0 \cos^2 \theta_1} = \frac{-\lambda_0 + 3\lambda_0 z_1^2}{2 + \lambda_0 - \lambda_0 z_1^2}, \quad (14)$$

where  $z_1 = \cos \theta_1$  is the component of the unit vector  $\hat{z}'$  along the  $z$  axis in the dilepton rest frame. From Eq. (13) and Eq. (14) we note that  $\mathcal{G}$  and  $\lambda_\theta$  are invariant under the rotation along the  $x$  and  $z$  axis, respectively.

Using the above results one can see that despite the nonlinear transformation of  $\lambda_\theta$ ,  $\lambda_{\theta\phi}$  and  $\lambda_\phi$  under rotations, the linear combination  $1 - \lambda_\theta - 4\lambda_\phi$  remains zero in all other rotated frames if it is zero in one particular frame, as was observed for specific rotations in [22]. If the combination is nonzero however, then its value will change under rotations, even around the  $y$  axis. From Eq. (9), it follows that the quantity  $1 - \lambda_\theta - 4\lambda_{\theta\phi}$  is not invariant under rotations along the  $y$  axis. On the other hand, the quantity,  $(1 - \lambda_\theta - 4\lambda_\phi)/(3 + \lambda_\theta)$ , is invariant under such rotations, namely

$$\frac{1 - \lambda_\theta - 4\lambda_\phi}{3 + \lambda_\theta} = 1 - 2\mathcal{F} = \frac{1 - \lambda_0 + 4\lambda_0 y_1^2}{3 + \lambda_0}. \quad (15)$$

Therefore, to examine the amount of the violation of the Lam-Tung relation, the quantity,  $(1 - \lambda_\theta - 4\lambda_\phi)/(3 + \lambda_\theta)$ , is preferred.

Often in the literature for the Drell-Yan process, another set of angular coefficients are considered:  $A_0$ ,  $A_1$ ,  $A_2$ , where

$$\begin{aligned} \frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2}(1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi. \end{aligned} \quad (16)$$

The Lam–Tung relation is then expressed as  $A_0 = A_2$ . The violation of the Lam–Tung relation,  $A_0 - A_2 = 2(1 - 2\mathcal{F})$ , is rotationally invariant around the  $y$  axis. On the other hand, the quantity  $\Delta_{LT} = 1 - A_2/A_0$  of [33] is not.

In conclusion, we have presented an intuitive derivation for rotation-invariant quantities for lepton angular distributions in Drell–Yan and vector quarkonium production. By expressing these quantities in terms of the  $\lambda_0$  and the  $x$ ,  $y$  and  $z$  components of the unit vector of the quark momentum in the dilepton rest frame, the invariant properties of these quantities become transparent. This approach offers a useful insight regarding the roles of  $\lambda_0$  and the acoplanarity of the partonic subprocesses in determining the applicability and values of these invariant quantities. This approach could also be extended to other hard processes, such as hadron pair production in  $e^+e^-$  annihilation, which is closely connected to the Drell–Yan and vector quarkonium production.

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